Axisymmetric Jet Forced by Fundamental and Subharmonic Tones

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A circular jet was excited simultaneously by two harmonically related tones. The results of this excitation on jet behavior are reported for three pairs of Strouhal numbers [$St(D) = f^*D/U_j$ = 0.2 and 0.4, 0.3 and 0.6, 0.4 and 0.8]. For each case, the initial phase difference between the two tones was varied in steps of 45 deg for one full cycle, and the amplitude of the fundamental and subharmonic tones was varied independently over the range of 0.1–7.0% of the jet exit velocity. Several results of this study agreed with other published findings, such as a critical amplitude of the fundamental being required for subharmonic augmentation and the initial phase difference being critical in determining whether the subharmonic is augmented or suppressed. In addition, the detailed documentation of several aspects of this phenomenon, measured in the same experimental facility in a controlled manner, brought out two important points that had eluded previous researchers. First, at high levels of the fundamental and subharmonic forcing amplitudes, the subharmonic augmentation is independent of the initial phase difference. Second, two-frequency excitation is indeed more effective than single-frequency excitation in jet mixing enhancement. Higher spreading rates seem to go along with higher subharmonic levels.

Nomenclature

A,B = constants
D = nozzle diameter
f = fundamental frequency
$f/2$ = subharmonic frequency
M = Mach number
r = radial distance
St = Strouhal number, $St(D) = fD/Uj$; $St(\Theta) = f\Theta/Uj$
t = time
U = mean velocity
$\tilde{u}$ = coherent component of velocity
w = frequency
x = axial distance
Θ = momentum thickness, $= \int_0^\infty \frac{U}{U_C}\left(1 - \frac{U}{U_C}\right) dr$
φ = phase difference

Subscripts
C = jet centerline
D = based on nozzle diameter
ex = excited
f = fundamental component
$f/2$ = subharmonic component
j = jet exit
o = initial
unex = unexcited
Θ = based on momentum thickness

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Introduction

BINDER and Favre-Marinet[1] undertook one of the first attempts at enhancing jet mixing by high-amplitude excitation. They performed a well-controlled unsteady forcing of a round jet using a rotating butterfly valve upstream of the nozzle exit. It is now well established that the behavior of an axisymmetric shear layer can be somewhat controlled by excitation at a single frequency near the preferred mode of the jet. This limitation arises because the instability wave can enhance mixing only if it continues to grow. When it no longer continues to grow, it is said to be saturated and its contribution to mixing enhancement ceases.

This phenomenon of nonlinearity and saturation has been documented by Crow and Champagne.[2] In a previous paper,[3] results were reported of an experiment that looked at the limit of jet mixing enhancement by single-frequency, plane wave excitation. Saturation was observed in all measured quantities of the jet evolution, i.e., the fundamental wave amplitude, the momentum thickness, the centerline mean velocity, and the turbulence intensity. The amplitude of the excitation at which the nonlinearities appeared was 0.3% of the jet velocity. The subsequent saturation occurred at an excitation amplitude of around 1% of the jet velocity, where the effect on the jet spreading rate was quantified by $U(\text{ex})/U(\text{unex}) = 0.85$ and $\Theta(\text{ex})/\Theta(\text{unex}) = 1.275$ at $x/D = 9$. These experimental results were also compared[3] with the predictions of a theoretical model by Mankbadi and Liu.[4]

The degree of jet spreading offered by single-frequency plane wave excitation may not seem attractive enough to pursue for practical applications. However, when the preferred mode frequency becomes neutrally stable, its subharmonic, which is then amplifying at its maximum rate, can be used to cause further mixing enhancement. The development of a subharmonic in a free shear layer has been observed by several researchers. An analysis was presented by Kelly[5] that showed that a mechanism exists for the generation of a subharmonic wave for a flow with a hyperbolic tangent velocity profile. Ho and Huang[6] later showed that the spreading rate of a mixing layer can be manipulated significantly by forcing near the subharmonic of the preferred frequency.

Furthermore, when the shear layer is excited simultaneously by the fundamental and subharmonic, an interaction could occur, leading to a large augmentation of the subharmonic amplitude. In the published literature, this phenomenon has been referred to as resonance or pairing. The axial extent and degree of control over shear-layer turbulence and spreading rate is increased if the jet is excited simultaneously by the fundamental and its subharmonic at optimum initial phase difference and amplitudes. Some aspects of this problem have been addressed by Ho and Huang,[6] Durbin and Mckinzie,[7] Arbey and Ffowcs-Williams,[8] Zhang et al.,[9] Mankbadi,[10] Monkewitz,[11] Cohen and Wygnanski,[12] Nikitopoulos and Liu,[13] Ng and Bradley,[14] and Hussain and Husain.[15] Arbey and Ffowcs-Williams[8] studied a circular jet simultaneously excited by two different harmonically related acoustic tones. They showed that control can be exercised on the harmonic generation process by varying the phase between the two signals. Zhang et al.[9] studied a plane mixing layer excited at fundamental and subharmonic frequencies and at various phase differences. The amplification rate of the subharmonic was found to depend strongly on the initial phase difference. Mankbadi[10] provided a theoretical analysis of the interaction between fundamental and subharmonic instability waves in a turbulent round jet. He used the energy integral approach to explain the subharmonic augmentation and mean flow manipulation in terms of energy exchanges between the fundamental, subharmonic, fine scale turbulence, and the mean flow.

The present work is a parametric study of the effect of simultaneous excitation, at the fundamental and subharmonic frequencies, on the behavior of a circular jet shear layer. The effect of the initial phase difference, Strouhal number pair, and amplitudes of the fundamental and subharmonic tones are investigated experimentally, and the initial conditions are carefully controlled and monitored. This work incorporates several novel features such as the use of high-amplitude excitation devices, which can provide a wide range of forcing conditions when used with complex waveform generators. The actual phase difference between the two waves was measured in the flow, as opposed to the more usual documentation of the phase difference in the input signal. These features have helped in producing data that lend new insights into the fundamentals of the two-frequency excitation problem and that will also stimulate future theoretical work. The study also shows the potential for two-frequency excitation to overcome the limitations of single-frequency excitation, with regard to mixing enhancement.

Experiment

Facility

The jet excitation facility (Fig. 1) consisted of a 30-in.-diam settling chamber, an excitation spool piece, and a convergent nozzle terminated with a 10-in.-long straight section having a 3.5-in. exit diam. Attached to the excitation spool piece were four Ling electropneumatic drivers (Model EPT 9B) operating at an air supply pressure of 40 lb/in.^{2} each and capable of producing low-frequency ($f < 1000$ Hz) complex waveforms with an acoustic power up to 4000 W.

The 40-psi air supplied to the Ling drivers was exhausted into the plenum tank by elbows that turned the flow upstream into the tank. Downstream of the elbows, a screen and honeycomb section conditioned the flow. The exhausted air passed through this flow conditioning and provided the airflow through the nozzle. Measurements made at the nozzle exit showed that the mean axial velocity profiles were uniformly "top hat" and unaffected by the air supply scheme. The turbulence intensity measured for the unforced jet at the nozzle exit was 1.5%, as opposed to 0.15% in previous experiments[3] where the Ling drivers were not used. Initially, this new system did pose a problem. If the test Mach number was reduced from 0.45 to 0.2, the pressure to the Ling drivers had to be reduced, which caused a drop in the acoustic output of the drivers, thus unacceptably changing the forcing levels at the jet exit. To provide a fine control on the operating Mach number independent of the forcing levels, four bleed valves were installed at the back end of the plenum (not shown in the schematic). The test Mach number could thus be reduced from 0.45 to 0.2 by bleeding off the excess air without affecting the acoustic output of the Ling drivers.

A specially fabricated boundary-layer trip ring was located 13 in. upstream of the nozzle exit, where the diameter of the contracting section was 5.12 in. The trip ring had 83 saw teeth that protruded 4.76 mm into the flow. The trip ring ensured that the jet exit boundary-layer characteristics were invariant with Mach number. The exit boundary layer was turbulent in all tests, similar to a full scale jet exhaust. A polynomial waveform synthesizer generated waveshapes from user supplied mathematical expressions. This signal was amplified by Altec Lansing power amplifiers and fed to the Ling drivers.

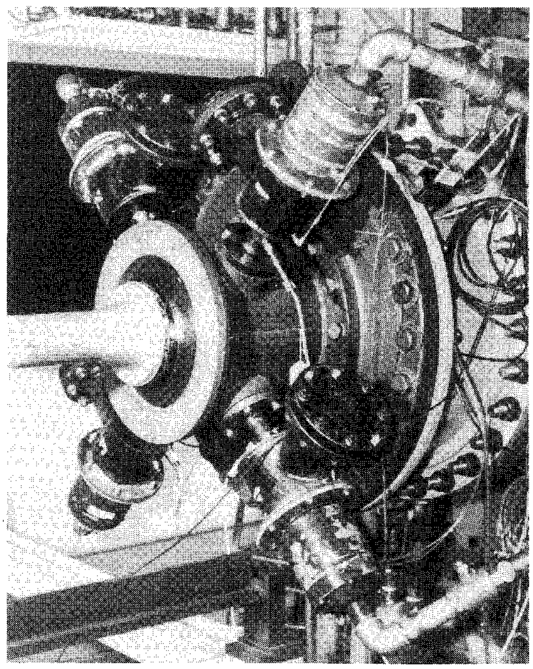


Fig. 1a Axisymmetric jet high-amplitude excitation rig.

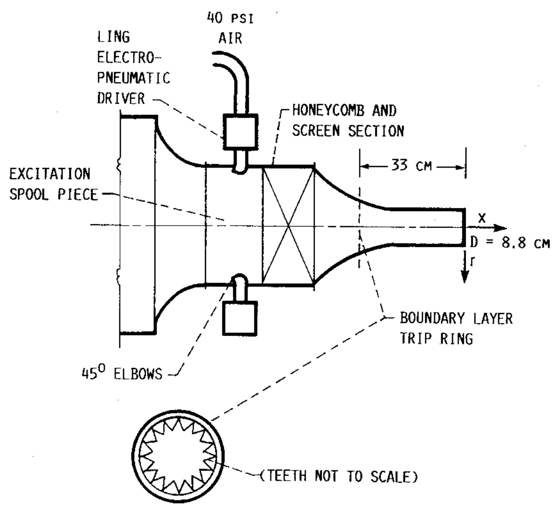


Fig. 1b Schematic of high-amplitude excitation facility.

Measurement Techniques

Measurements of mean and fluctuating velocities were made with hot-wire anemometers. The coherent motions were extracted by the phase-averaging technique, which takes the average of data points having the same phase with respect to a reference signal. Phase averaging rejects the background turbulence and educes the underlying periodic coherent component. In the present experiment, the reference signal was the forcing signal produced by the waveform generator. The phase average and phase angle difference measurements, as well as the spectrum analysis, were done using a B & K dual channel signal analyzer (Model 2025). A computer-controlled traversing mechanism was used for probe positioning.

The amplitudes and phase difference input by the waveform synthesizer to the Ling drives are completely different from those measured at the jet exit. Because the amplifier-driver system and the plenum tank resonance cause significant changes in the phase difference and amplitudes, depending on the frequencies and amplitudes input, documenting input conditions would be a poor substitute for the actual initial conditions. In the present work, the amplitudes of the two waves and the phase difference between them were measured in the flow at the jet exit (actual measurement station was 0.5 mm downstream of the jet exit) to constitute an accurate representation of the initial conditions.

Initial Conditions

The experiments were conducted for two jet exit velocities (M = 0.2 and 0.45). The turbulence intensity due to the random fluctuations measured at the jet exit centerline was measured to be 1.5% of the jet exit velocity. In order to keep the exit boundary layer from playing a parametric role in the experiment, a trip ring was located 13 in. upstream of the jet exit, which ensured that the exit momentum thickness shape factor and maximum root mean square fluctuations in the boundary layer remained constant for all test cases. The nozzle exit velocity profile was approximately top hat in shape, and the root mean square profile was uniform in the jet core at the nozzle exit.

The longitudinal velocity spectra measured at the center of the jet exit plane was contaminated by higher harmonics ($3/2f$, $2f$) from the high-amplitude Ling electropneumatic driver systems. When traced with downstream distance, the $3/2f$ and $2f$ components were much smaller than the f and the $f/2$ components and, therefore, were assumed not to interfere with the excitation process. For example, the peak magnitudes of the f and $f/2$ components were 7 and 20%, respectively, of the jet exit velocity, whereas the $3/2f$ and $2f$ peak magnitudes were 1 and 0.5%, respectively, of the jet exit velocity.

A parametric analysis of the effects of two-frequency excitation is complicated by a multitude of important parameters, including the initial phase difference and the absolute and relative forcing amplitudes of the two-frequency components. It is important to note that the initial phase difference φ_0 is defined for velocity and for sine waves

$$\tilde{u} = A \sin(2wt + \varphi_0) + B \sin(wt)$$

In this equation, the initial phase difference is the angle by which the fundamental coherent velocity leads the coherent subharmonic velocity. Three pairs of Strouhal numbers were studied. For each case, the initial phase difference between the two waves was varied in steps of 45 deg. The initial forcing level of the fundamental and subharmonic was also varied in the experiment. Table 1 shows a summary of the initial conditions for the various two-frequency cases.

Discussion of Results

Initial Phase Difference Effect

Growth of the Fundamental and Subharmonic on the Jet Centerline

The effect of varying the initial phase difference for the Strouhal number pair 0.2,0.4 is shown in Fig. 2. The initial phase difference between the two tones is denoted by φ_0. The fundamental forcing amplitude was 7% of the jet exit velocity and the subharmonic amplitude was 0.5%. The figure shows that the subharmonic $f/2$ is augmented to very high values for initial phase differences between 225 and 360(0) deg and suppressed for phase differences of 90 and 135 deg. Phase differences of 45 and 180 deg represent the intermediate states. At first glance, the fundamental f appears to be unaffected by the initial phase difference, but closer scrutiny reveals the kinks in its development, which are subtle reminders of the nonlinear interaction between the two waves. Kelly[5] explained this interaction between the two frequency components for parallel flows and showed that this could result in a transfer of energy between them. There have been other interpretations, such as the one by Stuart,[16] which deals with odd and even mode interactions of which the present two-frequency excitation is a special case. Liu[17] explained this dependence theoretically by interpreting the phase difference as being related to the angle between the stresses and the strains of the waves; the alignment of the stresses and the strains produces augmentation, whereas nonalignment produces suppression. Mankbadi et al.[18] applied Liu's explanation to the specific situation of an axisymmetric jet. Many other researchers have also pointed out the importance of the initial phase difference to the subharmonic growth.[9–11,14] Note

Table 1 Summary of initial conditions of two-frequency excitation

Strouhal number pair, $St(D)$	Fundamental forcing amplitude, $(\tilde{u}_f)/U_j$, %	Subharmonic forcing amplitude, $(\tilde{u}_{f/2})/U_j$, %	Ratio fund/sub	Phase difference, $(\varphi)_0$, deg
		M = 0.2		
0.2,0.4	3	3	1	0–360 (step 45)
0.3,0.6	3	3	1	0–360 (step 45)
		0.6	5	
		0.2	15	
	↓	0.1	30	
0.4,0.8	3	3	1	0–360 (step 45)
		M = 0.45		
0.2,0.4	7	0.47	15	0–360 (step 45)
	5.7	0.38	15	270
	3.4	0.23		
	2.8	0.19		
	2	0.13		
	1	0.067	↓	↓

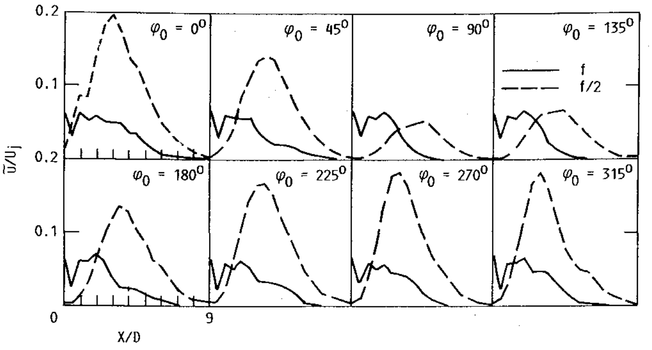


Fig. 2 Axial evolution of phase average unsteady velocity components on jet centerline for various initial phase differences: St = 0.2,0.4; $\tilde{u}_{f0}$ = 0.07 U_j; $\tilde{u}_{f/2_0}$ = 0.0045 U_j; M = 0.45.

that the extremely high level of the subharmonic (20% of the jet velocity) produced by the nonlinear interaction is much higher than can be attained by single-frequency excitation.

There are certain experimental difficulties in forcing the subharmonic by itself. The two-frequency signal at the jet exit is comprised of three contributions: 1) the independent $f/2$ signal, 2) the independent f signal, and 3) a portion of the f signal produced as a harmonic of the $f/2$. The $f/2$ alone data could not be taken easily because the introduction of $f/2$ automatically introduced some f. This problem was more pronounced at higher amplitudes of forcing and frequencies, which were close to resonant frequencies of the plenum tank. (Note that forcing f produced very little $2f$ as this was not in the range of the plenum resonance.)

However, to make the point that the two-frequency excitation is more effective than single-frequency excitation, one case (to be described later) was very carefully documented. This was possible because the $f/2$ amplitude was very low (0.2% of the jet velocity) where no harmonic f was produced.

Jet Centerline Axial Velocity

In the previous section, it was shown that, depending on the initial phase difference, the subharmonic can either be suppressed or augmented. Obviously, the result is going to affect the mean velocity and the random turbulence of the jet. The fundamental and subharmonic coherent velocities, the mean velocity, and the random turbulence are all coupled together and interact by energy exchanges. Figure 3a shows the jet centerline velocity plotted vs axial distance for the various phase differences. A steeper descent of the jet centerline velocity signifies a more rapid opening up of the jet plume. There is a very clear dependence between the subharmonic augmentation and the jet centerline velocity. Where the subharmonic is augmented to very high levels (Fig. 2), the jet centerline decays most rapidly. This is shown more specifically in Fig. 3b, where the velocity(excited)/velocity(jet) measured at x/D = 3.5 is plotted vs the initial phase difference. The unexcited and single-frequency (St = 0.4) excitation cases are shown for reference. As is readily apparent from the figure, two-frequency excitation is much more effective than single-frequency excitation at the fundamental frequency in reducing the jet centerline velocity.

Effect of Fundamental Forcing Level

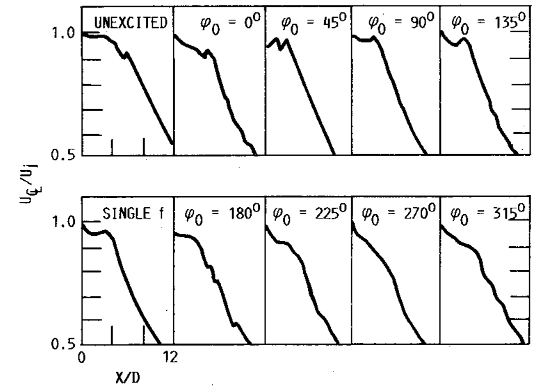
The effect of increasing the forcing level of the fundamental (while keeping the ratio of the fundamental-to-subharmonic levels constant at 15) on the growth of the two waves is shown in Fig. 4a for an initial phase difference of 270 deg. At low levels of fundamental forcing, no subharmonic augmentation is observed, but beyond a critical level of the fundamental, the subharmonic augmentation is seen. Kelly[5] has pointed out the existence of this critical level. As seen before, the jet centerline steady velocity is related to the subharmonic augmentation, and when the critical level of the fundamental is reached, the jet centerline velocity begins to drop, indicating enhanced mixing of the jet (Fig. 4b).

Effect of Strouhal Number Pair

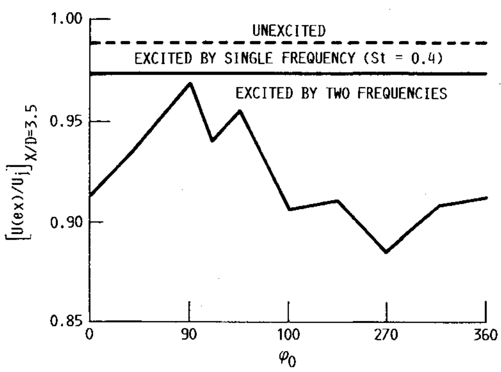
Figure 5 shows the development of the phase-averaged velocities of the fundamental and subharmonic for various initial phase differences. The initial level of the fundamental was equal to that of the subharmonic ($0.03U_j$). Data is shown for three pairs of Strouhal numbers (0.2,0.4; 0.3,0.6; and 0.4,0.8). The rationale for choosing these numbers was the preferred mode of the jet, which corresponds to a Strouhal number of 0.3 based on the jet diameter.[2] In the case of the 0.2,0.4 pair, the two frequencies straddle the preferred mode, and in the case of 0.3,0.6, the subharmonic frequency is the preferred mode, whereas in the last case 0.4,0.8, the subharmonic is slightly higher than the preferred mode.

The peak amplitude and axial location of the subharmonic in Fig. 5b do not depend highly on the initial phase difference. For a fixed initial phase difference, the subharmonic peak (pairing location) depends highly on the Strouhal number pair. The subharmonic peaks at x/D = 2.7 for the $St(D)$ = 0.2,0.4 case (Fig. 5a) and at x/D = 1.2 for $St(D)$ = 0.4,0.8 (Fig. 5c). In the shear-layer mode [$St(\Theta) = f\Theta/U_j$], the higher frequencies will saturate closer to the jet exit; thus, both the fundamental and the subharmonic peak closer to the jet exit for $St(D)$ = 0.4,0.8, whereas the curves peak farthest downstream for $St(D)$ = 0.2,0.4. The $St(D)$ = 0.3,0.6 case is between these two extremes.

Figures 6 show the phase difference between the two waves vs axial distance for the various initial phase differences, with the initial phase difference for each case subtracted out to make comparison easier. Linear behavior of the excited hydrodynamic modes might be most expected near the nozzle exit where the amplitudes are small and where negligible interaction has taken place. Initial phase difference should not



a) Axial variation of jet centerline velocity



b) Mean velocity ratio (excited/jet exit) at X/D = 3.5

Fig. 3 Two-frequency excitation influence on jet centerline velocity: St = 0.2,0.4; M = 0.45; $\tilde{u}_{f0}$ = 0.07 U_j; $\tilde{u}_{f/2_0}$ = 0.0045 U_j.

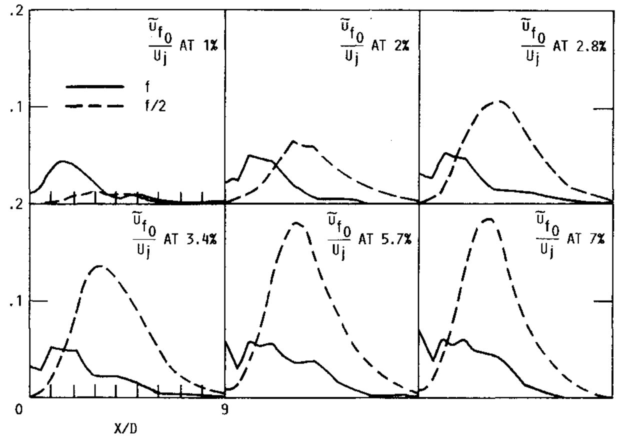


Fig. 4a Axial evolution of phase-averaged unsteady velocity components on the jet centerline for various initial levels of excitation: St = 0.2,0.4; φ_0 = 270 deg; $\tilde{u}_{f0}/\tilde{u}_{f/2_0}$ = 15; M = 0.45.

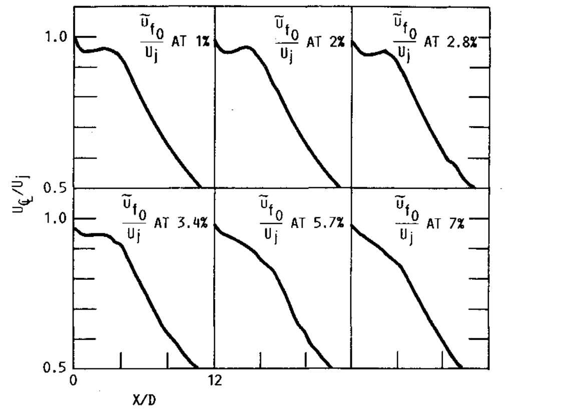


Fig. 4b Effect of increasing initial levels of excitation on the jet centerline mean velocity: St = 0.2,0.4; φ_0 = 270 deg; $\tilde{u}_{f0}/\tilde{u}_{f/2_0}$ = 15; M = 0.45.

be a factor for linear propagation of instability waves. The phase difference between the fundamental and subharmonic instabilities should change with distance as governed by their wavelength difference and the slight difference in phase velocity. As seen in Fig. 6, initial phase difference does not have much influence for small x/D, especially for the pair $St(D)$ = 0.2,0.4. Unfortunately, this result does not have anything to do with hydrodynamic instability propagation for small x/D. As clearly seen in Fig. 2, and less clearly in some parts of Figs. 5, the amplitude of the fundamental first drops at low x/D before finally increasing. This represents the falloff with distance of the acoustic field used for excitation until the rise of the hydrodynamic field dominates at higher x/D values.

A further complication is that the slope of the phase change is far too great to be explained by the propagation of two spherical waves at the fundamental and subharmonic frequencies. What is being observed at low x/D is the complex near-field evolution of the propagating and evanescent modes produced by the nozzle exit discontinuity, which cannot be further analyzed here. The phase difference at x/D = 0 is still considered a valid initial condition for comparing the experimental data, but it must be realized that hydrodynamic instabilities do not dominate the data until some distance downstream. The influence of initial phase angle on the data in Figs. 6 is an indicator of nonlinear interactions between the two-frequency modes. The spread of the results reaches a maximum at x/D values where subharmonic generation shown in Figs. 5 is at a maximum. Initial phase has the least influence for the pair $St(D)$ = 0.4,0.8, where subharmonic augmentation is minimal. This nonlinear two-mode interaction is currently being studied by Mankbadi et al.[18]

Figures 7 show the jet centerline velocity and the momentum thickness variation with streamwise distance for the unexcited jet and the three Strouhal number pairs. When the two-frequency excitation is applied, the jet centerline velocity drops and the spreading rate as quantified by the momentum thickness increases. At the high amplitudes used here, there is very little difference between the three Strouhal number pairs.

Variation of Subharmonic Forcing Level

Figures 8 show the development of the phase-averaged velocities of the fundamental and subharmonic for the Strouhal number pair 0.3,0.6. The initial forcing level of the fundamental is kept constant, whereas the forcing level of the subharmonic is varied from 0.1 to 3%, the latter having already been presented in Fig. 5b. In Figs. 8a–c, both the subharmonic peak and the axial location of the peak depend on the initial phase difference. Figure 5b shows that the peak attained by the subharmonic is not highly dependent on the initial phase difference when the initial subharmonic forcing level is high. The fact that, at high forcing amplitudes of the fundamental and subharmonic, the subharmonic is always augmented irrespective of the initial phase difference is not only useful but will have a favorable impact on the design of practical excitation devices. In Fig. 9, a summary of the cases from Figs. 8 and 5b, shows the subharmonic peak vs initial phase difference.

Figures 10 show the development of the phase difference with axial distance for the three cases of Fig. 8 and one case

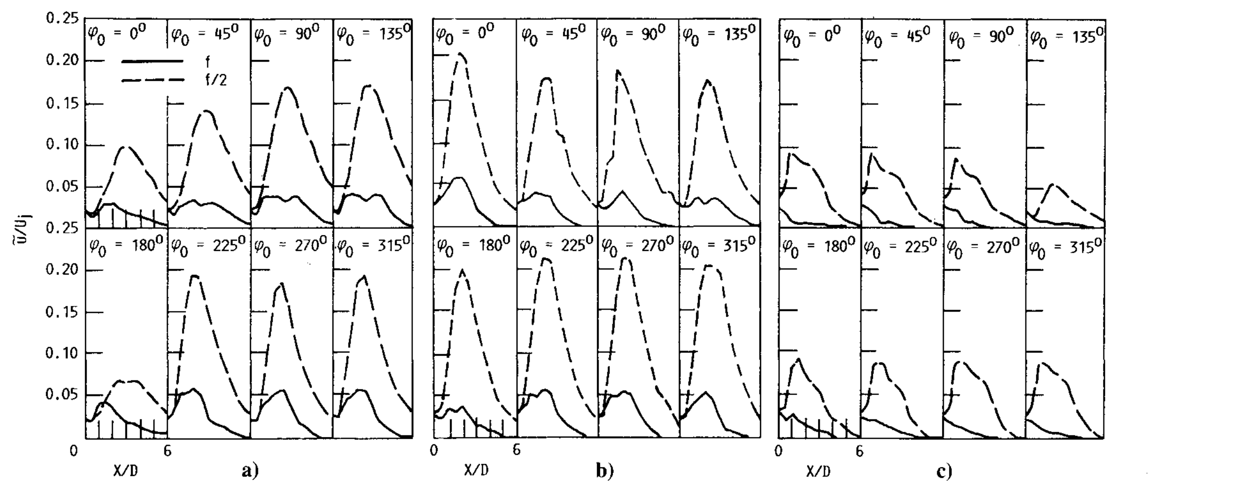


Fig. 5 Axial evolution of phase-averaged unsteady velocity components on the jet centerline for various initial phase differences and Strouhal number pairs ($\tilde{u}_{f_0}$ = $\tilde{u}_{f/2_0}$ = 0.03 U_j; M = 0.2): a) St = 0.2,0.4; b) St = 0.3,0.6; and c) St = 0.4,0.8.

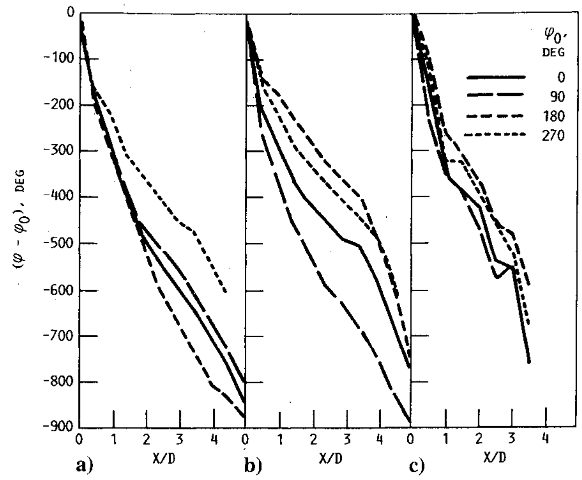


Fig. 6 Evolution of phase difference on the jet centerline for various initial phase differences and Strouhal number pairs ($\tilde{u}_{f_0} = \tilde{u}_{f/2_0} = 0.03\ U_j$; $M = 0.2$): a) $St = 0.2,0.4$; b) $St = 0.3,0.6$; and c) $St = 0.4,0.8$.

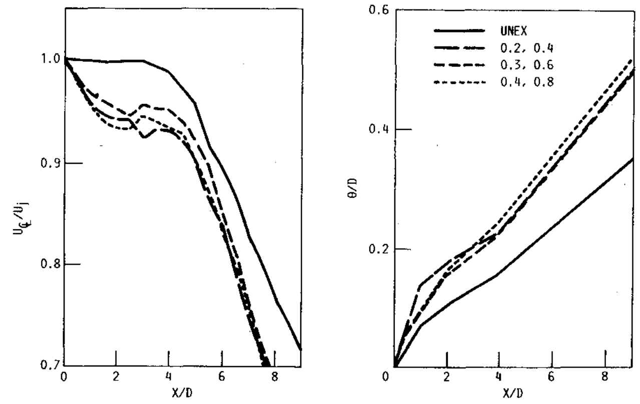


Fig. 7 Two-frequency excitation influence on jet centerline velocity and momentum thickness for various Strouhal number pairs: $\tilde{u}_{f_0} = \tilde{u}_{f/2_0} = 0.03\ U_j$; $M = 0.2$; $\varphi_0 = 270$ deg.

of Fig. 5b, but at phase difference intervals of 90 deg. As discussed in connection with Fig. 6, the initial region of the jet may be dominated by the acoustic near field, which is most evident for high excitation amplitude in Fig. 10d. The solid lines (phase difference = 0) in Figs. 10a and 10b might represent the closest approach to linear behavior since the subharmonic remains at very low levels (see Figs. 8a and 8b at phase difference = 0). As the initial phase angle and subharmonic excitation level are changed, the slopes of the curves decrease in the range of approximately $1 < x/D < 3$ (where the subharmonic amplitude is largest) probably due to the nonlinear interaction of the two modes. In this range, the phase angle difference is extremely sensitive to initial phase angle, especially at high excitation amplitude (Fig. 10d). The steep slope is again attained when the subharmonic amplitude has decayed substantially, which may indicate a change in the nonlinear mechanism or conceivably a return to linearity.

Jet Mixing Enhancement

The top row in Fig. 11 shows the development of the phase-averaged velocities along the jet centerline. The four cases represented are the fundamental alone, the subharmonic alone, both f and $f/2$ at phase = 180 deg, and, finally, both f and $f/2$ at phase = 0 deg. Though the waves do not grow to very large amplitudes individually, the two-frequency interaction causes the subharmonic to be augmented to very high levels (or suppressed), depending on the initial phase difference between the two waves. The second row shows the jet centerline velocity along with the unexcited case for comparison. The last row shows the momentum thickness development with axial distance. Very clearly under optimum conditions, two-frequency excitation is more effective than single-frequency excitation for jet mixing enhancement.

The jet centerline velocity and momentum thickness variation are both used as indicators of jet mixing enhancement. A drop in the centerline velocity indicates a higher jet spread for most cases. There is, however, a reversal in the jet centerline velocity trend between $x/D = 1$ and 3. This is not to be interpreted as a reversal of mixing. Because of the shortcomings of the jet centerline velocity as a mixing indicator, the momentum thickness is considered to be a better indicator of jet mixing enhancement. The jet centerline velocity is, however, retained as it shows a direct relationship between subharmonic augmentation and the eating up of the potential core of the jet. The high growth rate of the subharmonic wave causes it to extract energy from the mean flow, which causes the destruction of the potential core. Based on the theoretical work of Mankbadi and Liu,[4] the results of Figs. 11 can be interpreted as follow. The local regions of mean flow accel-

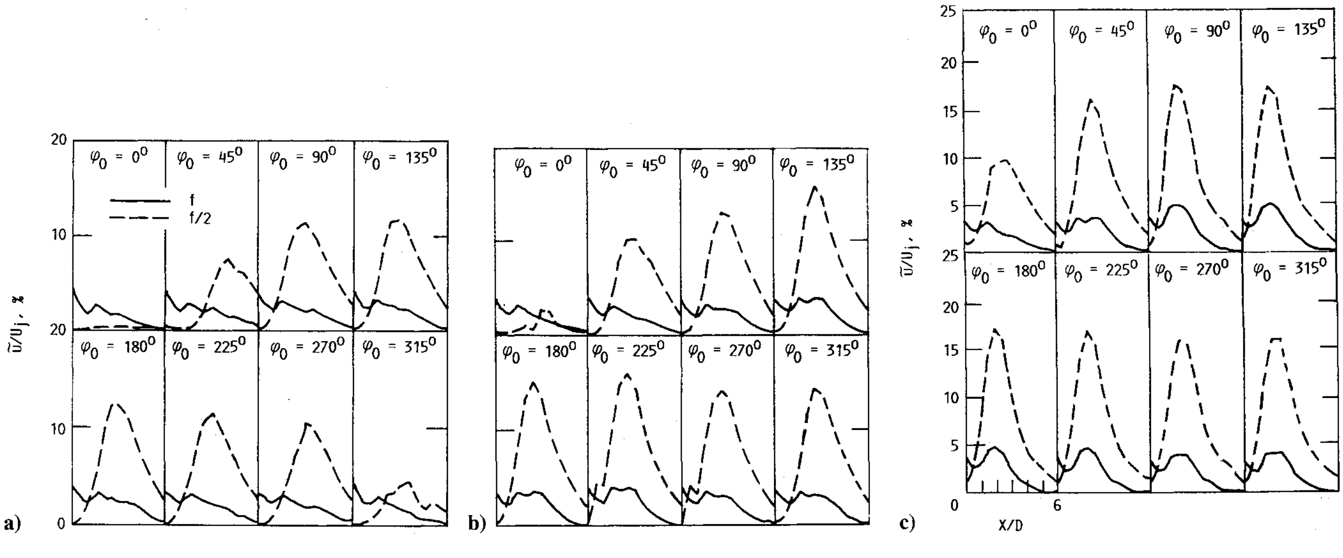


Fig. 8 Axial evolution of phase-averaged unsteady velocity components on the jet centerline for various initial phase differences and subharmonic forcing levels ($St = 0.3,0.6$; $\tilde{u}_{f_0} = 0.03\ U_j$; $M = 0.2$): a) $\tilde{u}_{f/2_0} = 0.001\ U_j$; b) $\tilde{u}_{f/2_0} = 0.002\ U_j$; and c) $\tilde{u}_{f/2_0} = 0.006\ U_j$.

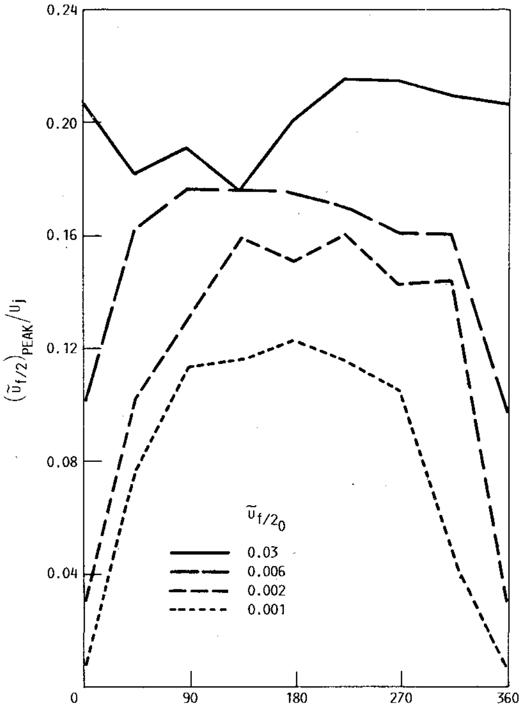


Fig. 9 Subharmonic peak variation with initial phase difference for various initial subharmonic forcing levels: St = 0.3,0.6; $\tilde{u}_{f_0}$ = 0.03 U_j; M = 0.2.

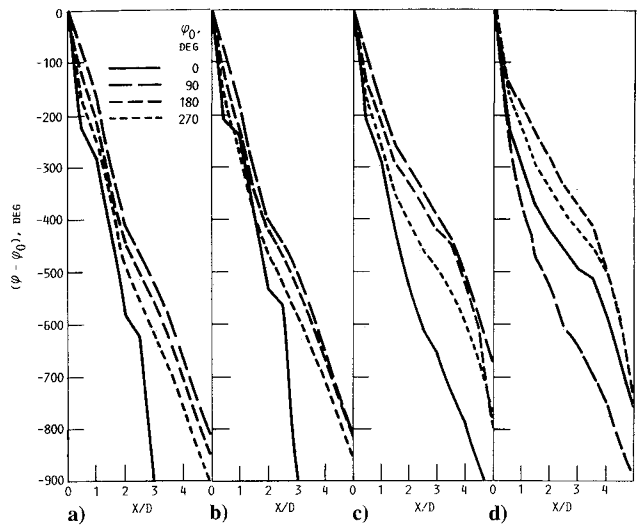


Fig. 10 Evolution of phase difference on the jet centerline for various initial phase differences and subharmonic forcing levels (St = 0.3,0.6; $\tilde{u}_{f_0}$ = 0.03 U_j; M = 0.2): a) $\tilde{u}_{f/2_0}$ = 0.001 U_j; b) $\tilde{u}_{f/2_0}$ = 0.002 U_j; c) $\tilde{u}_{f/2_0}$ = 0.006 U_j; and d) $\tilde{u}_{f/2_0}$ = 0.03 U_j.

eration might be attributed to modes evolving into their damped region where energy is pumped back into the mean flow from the oscillations, with accompanying decrease in the momentum thickness growth rate. The difference between single- and two-frequency optimum excitation results is most pronounced on the jet centerline velocity. The two-frequency case with phase difference equal to zero produces the same mixing as the single frequency excitation at the fundamental frequency. As seen in Fig. 11d, this two-frequency excitation suppresses the coherent subharmonic growth, and additional mixing from forced pairing does not result. Some random pairing may still occur, as might be expected in the single-frequency excitation case.

Time Traces

Figure 12 shows time traces at various locations on the jet centerline for the unexcited jet, two-frequency case phase = 180 deg, and two-frequency case phase = 0 deg. Figure 12a shows naturally occurring quasiperiodic waves at a period about the same as the subharmonic in Fig. 12b. Figure 12b shows the forced case at a phase difference of 180 deg and indicates that the frequency switches from 440 to 220 Hz (pairing) between x/D = 0.5 and 1 and the subharmonic peaks around x/D = 2. Further downstream, the random turbulence governs the time variations. Figure 12c shows the forced case at phase = 0.

At a phase difference of zero, the phase averaged measurements indicated a suppression of the subharmonic (Fig. 11). The time traces of Fig. 12c tell a more realistic story. The merging of vortices (frequency switching) occurs intermittently in space and time. At x/D = 2, time traces are shown at two different instants; one trace shows a strong velocity component at 220 Hz and the second trace shows a weak component at 440 Hz. When time is averaged and plotted, it appears that the subharmonic is suppressed (Fig. 11), although, in reality, it is augmented for short periods of time followed by longer periods of suppression. Another point that can be made from Fig. 12b is the forced pairing that was obtained by the two-frequency forcing in a jet with an initially turbulent boundary layer. Zaman and Hussain[19] showed that, for a forced jet with a laminar exit boundary layer, vortex pairing is regular in space and time, whereas in a jet with a turbulent initial condition, it becomes intermittent in space and time. Fiedler and Mensing[20] showed that, for a plane turbulent shear layer, regular vortex pairing can be obtained by high-amplitude, single-frequency excitation. Figures 5, 8, and 12b show that under the right conditions stable pairing can be induced by two-frequency excitation for a jet with a turbulent initial condition.

Ng and Bradley[14] have observed that when the jet is forced at more than one frequency the vortex merging process is highly localized and can be frozen visually in space by adjusting the phase and frequency of the strobe light, whereas this is not so easily accomplished for the single-frequency case. However, since they do not document the jet exit boundary-layer characteristics for their experiments, no meaningful comparison can be made with the present work.

Radial Distribution of Unsteady Velocities

Figure 13 shows the radial distribution of unsteady axial velocity, which is phase averaged at the fundamental and subharmonic frequencies. Also shown in the figure is the radial distribution of the mean velocity. The measurements are taken at x/D = 2, where the subharmonic peaks. It is at this location that the vortex pairing processes can be observed (e.g., Moore[21]). As the amplitude of the subharmonic is much larger than the fundamental, both along the axis and along the radius, it seems that the interaction is only weakly nonlinear and, therefore, a locally linear parallel flow instability theory will adequately provide the transverse distribution of velocity for both fundamental and subharmonic. Strange and Crighton[22] have shown this to be true for single-frequency cases. Here, data on the radial distribution are shown for the two-frequency case, which justifies the shape assumptions used by some researchers in some forms of the energy equation that governs the nonlinear streamwise evolution of the distribution amplitudes (e.g., Mankbadi[10]).

Concluding Remarks

The effect of exciting an axisymmetric jet simultaneously at fundamental and subharmonic frequencies was parametrically studied. Three Strouhal number pairs were studied (0.2,0.4; 0.3,0.6; 0.4,0.8) and the initial phase difference between the two waves was varied in steps of 45 deg. The effect

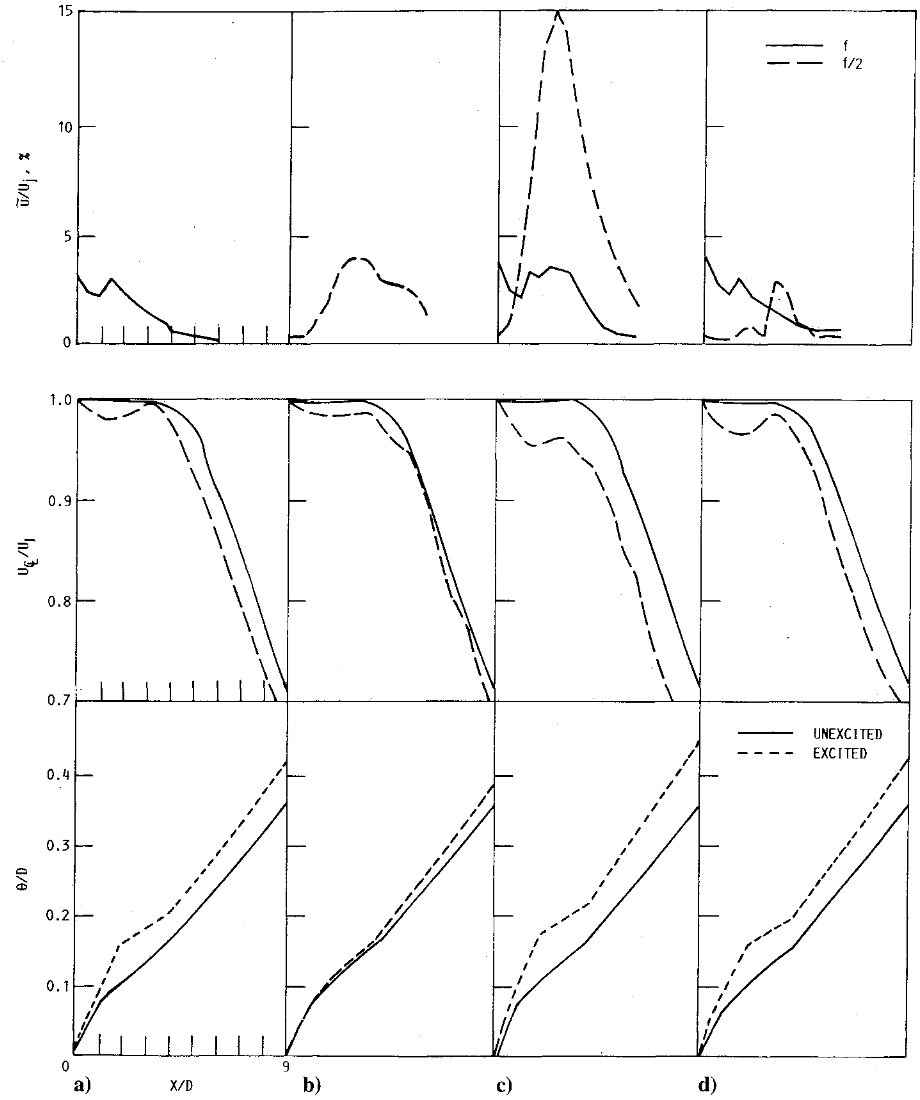


Fig. 11 Subharmonic augmentation effects on jet centerline velocity and momentum thickness ($M = 0.2$): a) $St = 0.6$; $\tilde{u}_{f/2_0} = 0.03\ U_j$; b) $St = 0.3$; $\tilde{u}_{f/2_0} = 0.002\ U_j$; c) $St = 0.3,0.6$; $\varphi_0 = 180$ deg; $\tilde{u}_{f_0} = 0.03\ U_j$; $\tilde{u}_{f/2_0} = 0.002\ U_j$; and d) $St = 0.3,0.6$; $\varphi_0 = 0$ deg; $\tilde{u}_{f_0} = 0.03\ U_j$; $\tilde{u}_{f/2_0} = 0.002\ U_j$.

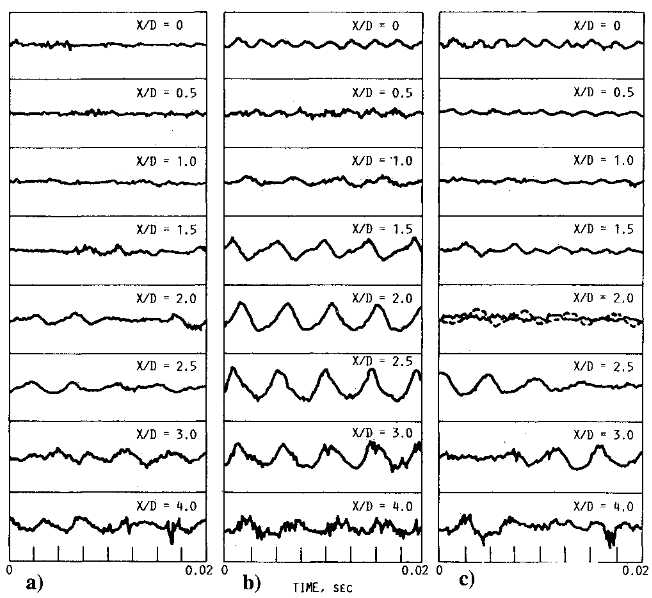


Fig. 12 Time traces of the velocity signal at various axial locations on the jet centerline ($M = 0.2$): a) Unexcited natural jet; b) Excited at $St = 0.3,0.6$; $\tilde{u}_{f_0} = 0.03\ U_j$; $\tilde{u}_{f/2_0} = 0.002\ U_j$; $\varphi_0 = 180$ deg; and c) Excited at $St = 0.3, 0.6$; $\tilde{u}_{f_0} = 0.03\ U_j$; $\tilde{u}_{f/2_0} = 0.002\ U_j$; $\varphi_0 = 0$ deg.

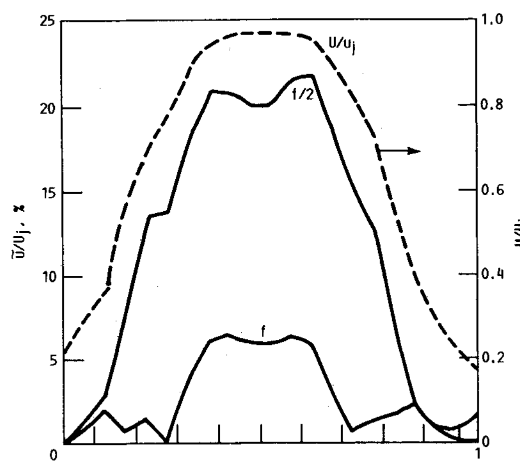


Fig. 13 Radial shapes of fundamental and subharmonic waves: $St = 0.3,0.6$; $\tilde{u}_{f_0} = 0.03\ U_j$; $\tilde{u}_{f/2_0} = 0.03\ U_j$; $M = 0.2$; $X/D = 2$; $\varphi_0 = 0$.

of varying the initial forcing levels was also studied. The following was found.

1) At high amplitudes of the fundamental and subharmonic forcing levels, the subharmonic augmentation and the axial location of the peak are independent of the initial phase difference. This finding will have a very favorable impact on the design of practical excitation devices.

2) Two-frequency excitation is indeed more effective than single-frequency excitation in jet mixing enhancement. The mixing is quantified by 1) jet centerline velocity, which shows the eating up of the potential core; 2) the momentum thickness, which shows the jet spreading rate; and 3) phase-averaged coherent velocities, which indicate the role of large scale coherent motions in mixing enhancement.

References

[1]Binder, G., and Favre-Marinet, M., "Mixing Improvement in Pulsating Turbulent Jets," *Fluid Mechanics of Mixing; Proceedings of the Joint Meeting*, edited by E. M. Uram and V. W. Goldschmidt, American Society of Mechanical Engineers, New York, 1973, pp. 167–172.

[2]Crow, S. C., and Champagne, F. H., "Orderly Structure in Jet Turbulence," *Journal of Fluid Mechanics*, Vol. 48, Part 3, Aug. 1971, pp. 547–591.

[3]Raman, G., Rice, E. J., and Mankbadi, R. R., "Saturation and the Limit of Jet Mixing Enhancement by Single Frequency Plane Wave Excitation: Experiment and Theory," *National Fluid Dynamics Congress, 1st*, Pt. 2, AIAA, New York, 1988, pp. 1000–1007; also "Collection of Technical Papers," AIAA 88-3613 CP.

[4]Mankbadi, R., and Liu, J. T. C., "A Study of the Interactions Between Large-Scale Coherent Structures and Fine Grained Turbulence in a Round Jet," *Philosophical Transactions of the Royal Society of London, Series A*, Vol. 298, No. 1443, 1981, pp. 541–602.

[5]Kelly, R. E., "On the Stability of an Inviscid Shear Layer Which is Periodic in Space and Time," *Journal of Fluid Mechanics*, Vol. 27, Part 4, March 1967, pp. 657–689.

[6]Ho, C. M., and Huang, L. S., "Subharmonics and Vortex Merging in Mixing Layers," *Journal of Fluid Mechanics*, Vol. 119, June 1982, pp. 443–473.

[7]Durbin, P., and Mckinzie, J. M., private communication, Sept. 1985.

[8]Arbey, H., and Ffowcs-Williams, J. E., "Active Cancellation of Pure Tones in an Excited Jet," *Journal of Fluid Mechanics*, Vol. 149, Dec. 1984, pp. 445–454.

[9]Zhang, Y. Q., Ho, C. M., and Monkewitz, P., "The Mixing Layer Forced by Fundamental and Subharmonic," *Laminar-Turbulent Transition*, edited by V. V. Kozlov, Springer-Verlag, New York, 1985, pp. 385–396.

[10]Mankbadi, R. R., "On the Interaction Between Fundamental and Subharmonic Instability Waves in a Turbulent Round Jet," *Journal of Fluid Mechanics*, Vol. 160, Nov. 1985, pp. 385–419.

[11]Monkewitz, P. A., "Subharmonic Resonance, Pairing and Shredding in the Mixing Layer," *Journal of Fluid Mechanics*, Vol. 188, March 1988, pp. 223–252.

[12]Cohen, J., and Wygnanski, I., "The Evolution of Instabilities in the Axisymmetric Jet. Part 2. The Flow Resulting from the Interaction Between Two Waves," *Journal of Fluid Mechanics*, Vol. 176, March 1987, pp. 221–235.

[13]Nikitopoulos, D. E., and Liu, J. T. C., "Nonlinear Binary Mode Interactions in a Developing Mixing Layer," *Journal of Fluid Mechanics*, Vol. 179, June 1987, pp. 345–370.

[14]Ng, T. T., and Bradley, T. A., "Effect of Multi-Frequency Forcing on the Near Field Development of a Jet," AIAA Paper 87-0054, Jan. 1987.

[15]Hussain, A. K. M. F., and Husain, H. S., "Passive and Active Control of Jet Turbulence," *Turbulence Management and Relaminarization: Proceedings of the IUTAM Symposium, Bangalore, India 1987*, edited by H. W. Liepmann and R. Narashima, Springer-Verlag, New York, 1988, pp. 445–458.

[16]Stuart, J. T., "Nonlinear Effects in Hydrodynamic Stability," *Proceedings, 10th International Congress on Applied Mechanics*, edited by F. Rolla and W. T. Koiter, Elsevier, New York, 1962, pp. 63–97.

[17]Liu, J. T. C., "Interactions Between Large-Scale Coherent Structures and Fine-Grained Turbulence in Free Shear Flows," *Transition and Turbulence*, edited by R. E. Meyer, Academic, New York, 1980, pp. 167–214.

[18]Mankbadi, R. R., Raman, G., and Rice, E. J., "On the Conditions for Resonance Interactions of Instability Waves in the Axisymmetric Jet," NASA TM 101477, March 1989.

[19]Zaman, K. B. M. Q., and Hussain, A. K. M. F., "Vortex Pairing in a Circular Jet Under Controlled Excitation. Part 1. General Jet Response," *Journal of Fluid Mechanics*, Vol. 101, Dec. 1980, pp. 449–491.

[20]Fiedler, H. E., and Mensing, P., "The Plane Turbulent Shear Layer with Periodic Excitation," *Journal of Fluid Mechanics*, Vol. 150, Jan. 1985, pp. 281–309.

[21]Moore, C. J., "The Role of Shear Layer Instability Waves in Jet Exhaust Noise," *Journal of Fluid Mechanics*, Vol. 80, April 1977, pp. 321–367.

[22]Strange, P. J. R., and Crighton, D. G., "Spinning Modes on Axisymmetric Jets, Part I," *Journal of Fluid Mechanics*, Vol. 134, Sept. 1983, pp. 231–245.